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SHORTER COMMUNICATIONS

ON THE EXTENSION TO NON-NEWTONIAN FLUIDS OF MANGLER'S RECTILINEARIZATION OF AXISYMMETRIC LAMINAR BOUNDARY-LAYER FLOWS

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NOMENCLATURE

a, coefficient;
n, exponent;
p, local static pressure;
r, local radius of axisymmetric body;
s, dummy;
u, local velocity component along the x-direction;
v, local velocity component normal to the body surface;
x, minimal distance along body surface from forward stagnation point;
y, local normal distance to the body surface;
μ, effective coefficient of dynamic viscosity;
ρ, density;
τ_{yx}, local viscous stress in the x-direction on a surface parallel to the local body surface;
ϕ, function of x.

A barred variable is the transformation of the corresponding unbarred variable.

HAYASI [1] has observed that Mangler's well-known transformation of the boundary-layer equations for steady axisymmetric laminar flows into those for steady rectilinear laminar flows is readily generalized so as to be applicable to the so-called Ostwald–de Waele power-law fluids. The main purpose of this communication is to point out that such a generalization is inherent for a still broader class of fluids.

Starting with the mild restriction that the fluid's viscous stress characteristics are such as to not negate the applicability of Prandtl's boundary-layer hypotheses or equations, the governing equations for steady axisymmetric boundary-layer flow may be written as equations (1) and (2).

$$\partial(ru)/\partial x + \partial(rv)/\partial y = 0 \tag{1}$$

$$\partial(u\,\partial u/\partial x + v\,\partial u/\partial y) = -\,\mathrm{d}p/\mathrm{d}x + \partial\tau_{yx}/\partial y. \tag{2}$$

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Upon substitution therein of the generalization of Mangler's transformations of equations (3), (4) and (5),

$$\bar{x} = \int_0^x r\phi\,\mathrm{d}x \tag{3}$$

$$\bar{y} = ry \tag{4}$$

$$\text{and } \phi\bar{v} = v + (\bar{y}u/r^2)\,\mathrm{d}r/\mathrm{d}x \tag{5}$$

where ϕ is not a function of y, equations (1′) and (2′) are found.

$$\partial u/\partial\bar{x} + \partial\bar{v}/\partial\bar{y} = 0 \tag{1′}$$

$$\rho\phi(u\,\partial u/\partial\bar{x} + \bar{v}\,\partial u/\partial\bar{y}) = -\,\phi\,\mathrm{d}p/\mathrm{d}\bar{x} + \partial\tau_{yx}/\partial\bar{y}. \tag{2′}$$

Clearly all that is necessary for a successful rectilinearization of the axisymmetric boundary-layer equations is for the viscous stress relation of equation (6) to be true.

$$\tau_{yx} = \phi\,\bar{\tau}_{\bar{y}\bar{x}}. \tag{6}$$

Thus not only are Newtonian and Oswald–de Waele fluids allowed but also, for example, the first two more general non-Newtonian fluid representations discussed in reference [2] in connection with the occurrence of similarity solutions, i.e. fluids whose effective viscosity may be written either as equation (7) or as equation (8).

$$\mu = a\,|\partial^s u/\partial y^s|^{n-1} \tag{7}$$

$$\mu = \Sigma\, a_s\,|\partial^s u/\partial y^s|^{(n-1)/s} \tag{8}$$

It follows that similar axisymmetric laminar boundary-layer flows occur for a broad class of fluid models.

REFERENCES

1. N. HAYASI, Correlation of two-dimensional and axisymmetric boundary-layer flows for purely viscous non-Newtonian fluids, *AIAA Jl* **3**, 532–533 (1965).
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